January 29, 2016	Math 2L03	Dr. Prayat Poudel

Test 1	50 Minutes

Full Name	Student I.D.
Solution	

THIS EXAMINATION PAPER INCLUDES **7** PAGES AND **4** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

INSTRUCTIONS: No aids except the standard Casio fx991 calculator are permitted.

Problem	Points	Score
1	40	
2	30	
3	15	
4	15	
Total:	100	

1. (40 points) The stated goal is to forecast annual sales for all new stores, based on store size. To examine the relationship between the store size in square feet and its annual sales, a sample of 5 stores was selected. Table below summarizes the results for these 5 stores.

Square Feet (in Thousands) x	Annual Sale (in Million Dollars) y
1	2
3	3
5	4
6	7
8	8

(a) (10 points) Compute the sum-of-square errors (SSE) for the linear model $y = x + 1$.

x	y	$\hat{y} = x+1$	Residual $y-\hat{y}$	Residual^2 $(y-\hat{y})^2$
1	2	2	0	0
3	3	4	-1	1
5	4	6	-2	4
6	7	7	0	0
8	8	9	-1	1

$$SSE = 0+1+4+0+1$$
$$= 6$$

(b) (20 points) Use the information to find the linear regression model for annual sales (in millions of dollars) as a function of square feet(in thousands), and also the coefficient of correlation r. (Round coefficient to 2 decimal places)
(YOU WILL NEED TO SHOW YOUR WORK TO RECEIVE FULL CREDIT)

	x	y	x^2	y^2	xy
	1	2	1	4	2
	3	3	9	9	9
	5	4	25	16	20
	6	7	36	49	42
	8	8	64	64	64
Sum	23	24	135	142	137

$n = 5$

$y = mx + b$

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} = \frac{5(137) - 23 \cdot 24}{5(135) - (23)^2} = \frac{133}{146} \approx 0.91$$

$$b = \frac{\Sigma y - m\Sigma x}{n} = \frac{24 - \frac{133}{146} \cdot 23}{5} \approx 0.61$$

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\sqrt{n(\Sigma y^2) - (\Sigma y)^2}} = \frac{133}{\sqrt{146} \cdot \sqrt{134}} \approx 0.95$$

(c) (5 points) According to the model, how much square feet (in thousands) would the company need in order for its net annaul income to be \$10 million.

$$y = 0.91x + .61$$

$$\frac{10 - .61}{.91} = x \Rightarrow x = 10.31 \text{ sq. feet (in thousands)}$$

(d) (5 points) What does the value of r suggest about the relationship of annual sales to the store size?

$r > 0$, means positive slope and $r \approx 0.95$ is close to 1, the points lie nearly along a straight line.

2. (30 points) Use Gauss-Jordan (row) reduction to solve the given system of equations.

$$x + y + z = 2$$
$$\frac{x}{2} - \frac{y}{4} + \frac{z}{2} = -\frac{1}{2}$$
$$3x + y + 2z = 3$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ \frac{1}{2} & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} \\ 3 & 1 & 2 & 3 \end{bmatrix} \begin{array}{l} \\ 4R_2 \\ \\ \end{array} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -2 \\ 3 & 1 & 2 & 3 \end{bmatrix} \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \longrightarrow$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -3 & 0 & -6 \\ 0 & -2 & -1 & -3 \end{bmatrix} \begin{array}{l} \\ -\frac{1}{3}R_2 \\ \\ \end{array} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -1 & -3 \end{bmatrix} \begin{array}{l} R_1 - R_2 \\ \\ R_3 + 2R_2 \end{array} \longrightarrow$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{array}{l} R_1 + R_3 \\ \\ \\ \end{array} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{array}{l} \\ \\ -1R_3 \end{array}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \begin{array}{l} x = 1 \\ y = 2 \\ z = -1 \end{array}$$

3. (15 points) A manufacturer produces a battery-powered calculator and a solar model. The battery-powered model requires **10** min of electronic assembly and the solar model **15** min. There are **450** min of assembly time available per day. Both models require **8** min for packaging, and **280** min of packaging time are available per day. If the manufacturer wants to use all the available time, how many of each unit should be produced per day?

b = # of battery powered

s = # of solar model

$10b + 15s = 450$

$8b + 8s = 280 \Rightarrow b+s = 35 \Rightarrow b = 35 - s$

$10(35-s) + 15s = 450$

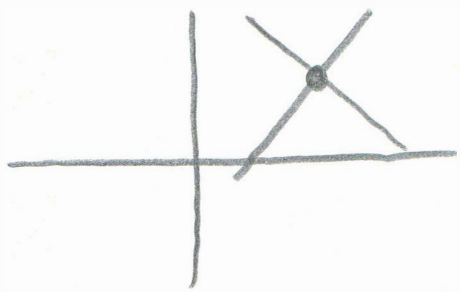
$\Rightarrow 350 - 10s + 15s = 450$

$\Rightarrow 5s = 100 \Rightarrow \underline{s = 20}$

$\underline{b = 35 - 20 = 15}$

4. (15 points) Given a system of 2 linear equations in 2 unknowns, describe all possible types of solutions to the system of equations. Give an example for each case.

Unique Solution



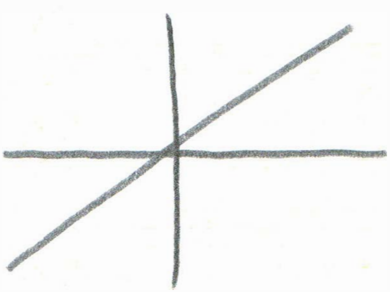
The two linear equations intersect at a single point.

Ex $y = x - 3$ $m = 1$

$y = -x + 5$ $m = -1$

Infinitely many solutions

Redundant system



Two linear equations represent the same line.

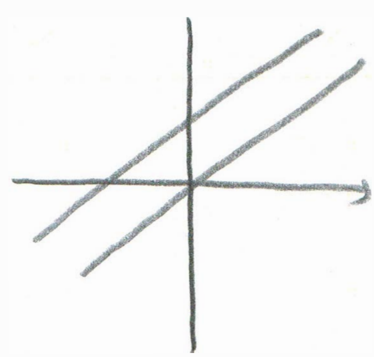
$y = x + 4$

$3y = 3x + 12$

No solution

Inconsistent System

Two lines are parallel (same slope) but different y-intercepts



$y = x + 1$

$y = x$